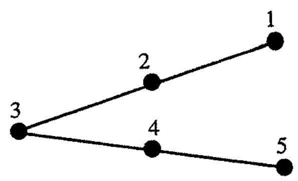
**INTRODUCTION TO MATROID THEORY**
**SUGGESTED EXERCISES**

1. Compute the Tutte polynomial of the following matroids using the Tutte-Grothendieck recursion.
   (a) The matroid with $k$ elements, all of which are loops (this is $U_{0,k}$.
   (b) The matroid with $k$ elements, all pairwise parallel (this is $U_{1,k}$).
   (c) Using part (a) and (b) it will be easier to compute the Tutte polynomial of




2. Let $M_1$, $M_2$ be two matroids. Prove that

$$T_{M_1 \oplus M_2}(x, y) = T_{M_1}(x, y) T_{M_2}(x, y).$$

   One possibility to do this is as follows. Let $E_1, E_2$ be the ground sets of $M_1$ and $M_2$. Without loss of generality suppose that $E_1$ and $E_2$ are disjoint. Then $M_1 \oplus M_2$ has ground set $E_1 \cup E_2$. Also let $\mathrm{rk}_1$, $\mathrm{rk}_2$, $\mathrm{rk}_\oplus$ denote the rank function of $M_1$, $M_2$, $M_1 \oplus M_2$.
   (a) Prove that, for all $X \subseteq E_1 \cup E_2$,

$$\mathrm{rk}_\oplus(X) = \mathrm{rk}_1(X \cap E_1) + \mathrm{rk}_2(X \cap E_2)$$

   (Hint: remember the description of the independent sets of the direct sum from the very first exercises!)
   (b) Use (a) in order to prove the desired equality.