INTRODUCTION TO MATROID THEORY
SUGGESTED EXERCISES
(CLASS OF MARCH 29TH, 2018)

1. A standard definition for geometric lattices is the following:
   "A finite lattice L is geometric if
   (a) *it is ranked (meaning: it possesses a rank function),*
   (b) *it is atomic (meaning: every element $x \in L$ can be expressed as a join of atoms)*[1],
   (c) *for all $x, y \in L$, if $x, y \gtrdot x \wedge y$, then $x \vee y \gtrdot x, y$.* "
   Prove that this definition is equivalent to the one given in class.

2. Let $\mathcal{B}$ be the set of bases of a matroid on the ground set E. Prove that the following property holds:
   (B2)* For every $B_1, B_2 \in \mathcal{B}$ and every $y \in B_2 \setminus B_1$ there is $x \in B_1$ such that $(B_1 \setminus \{x\}) \cup \{y\} \in \mathcal{B}$.

[1] More precisely: for every $x \in L$ there is $\ell \in \mathbb{N}$ and atoms $a_1, \ldots, a_\ell \in A(L)$ such that $x = a_1 \vee a_2 \vee \ldots \vee a_\ell$