

INTRODUCTION TO MATROID THEORY
SUGGESTED EXERCISES
(CLASS OF MARCH 8TH, 2018)

1. Prove that $U_{2,4}$ is not graphic and that it is not representable over the field \mathbb{F}_2 with two elements.

In the following let M be a matroid over the ground set E .

2. Let B be a basis of M and choose $e \in E \setminus B$. Prove:
 - there is a unique circuit contained in $B \cup \{e\}$. This circuit is called *the basic circuit* of e with respect to B and denoted by $C(B, e)$.
 - for each $f \in E$, $(B \setminus \{f\}) \cup \{e\}$ is a basis of M if and only if $f \in C(B, e)$.
3. Let rk be the rank function of M and choose $A \subseteq E$. Define a new function $\text{rk}_{/A} : 2^{E \setminus A} \rightarrow \mathbb{N}$ by setting, for all $Y \subseteq E \setminus A$,

$$\text{rk}_{/A}(Y) := \text{rk}(Y \cup A) - \text{rk}(A).$$

Prove that $\text{rk}_{/A}$ is the rank function of a matroid on the ground set $E \setminus A$. This matroid is called the *contraction* of A in M and is denoted by M/A .