Emanuele Delucchi
BeNeFri, Fall 2020

WORKSHEET 2

Recall:

- We have set up an **on-line meeting on October 29, at 15:30**, in order to discuss any questions related to modules 1 and 2, and in particular any assignments you find difficulty with.
- There might be some ("non-starred") assignments from Worksheet 1 that you have left for after Module 2.

Please let me anyway know in advance of the zoom-meeting if there is anything special you would like me to discuss in that occasion. This helps me prepare for it and gauge what, if anything, I might adjust in preparing the remainder of the lecture.

1. Reading

(i) Read from the original source the part of the proof of Theorem 1.1 (Lecture Notes #2) that we did not discuss yet (this is §4.3 in the Gel'fand-Goresky-MacPherson-Serganova paper I included in the literature folder).

(ii) Read from the Lecture Notes # 2 the proofs of Lemma 1.8 (revised); Lemma 2.1, Lemma 2.10 and Corollary 2.11(omitted in the lecture); Proposition 2.4 (partly omitted in the lecture),

(iii) Read the proof of Theorem 1.10 (revised) from Lecture Notes #2, now with an illustrative example, and if needed watch the explicatory video.

2. Solving

(i) Prove Lemma 2.5 in the Lecture Notes #2.

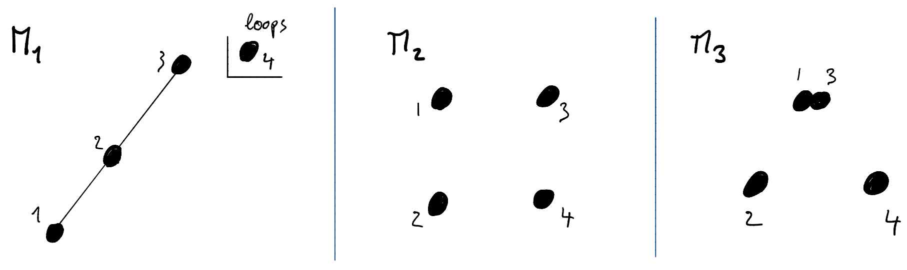
(ii) For all examples given below draw the matroid polytope, determine its facets that intersect the interior of the ambient, and compare them with the flacets.

(a) The uniform matroids $U_{2,4}$ and $U_{2,3}$

(b) The matroid on [4] with circuit set $\{\{1,3\},\{2,4\}\}$

(c) The uniform matroid $U_{3,4}$

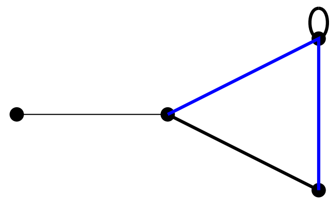
(d) The three matroids given by the diagrams below:



Turn page!

(iii) Prove the following statement: A matroid $M = (E, \mathcal{I})$ is disconnected (i.e., $c(M) > 1$) if and only if there is a subset A, $\emptyset \subsetneq A \subsetneq E$, such that $M = M[A] \oplus M[E \setminus A]$. [1]

(iv) Let G be the graph depicted below and consider the matroids $M(G)/X$ and $M(G)/Y$, where X is the set of blue edges, and Y is the set of all bold-faced edges. Find graphs G', G'' such that $M(G') \simeq M(G)/X$ and $M(G'') \simeq M(G)/Y$.



Let now M be an arbitrary matroid on the ground set E and let $A \subseteq E$. What is the difference between M/A and $M/\operatorname{cl}(A)$?

[1]Hint: choose $e \in E$ and let A be the equivalence class of e with respect to the equivalence relation $\sim_M$ defined in §2.1. of the Lecture notes #2.