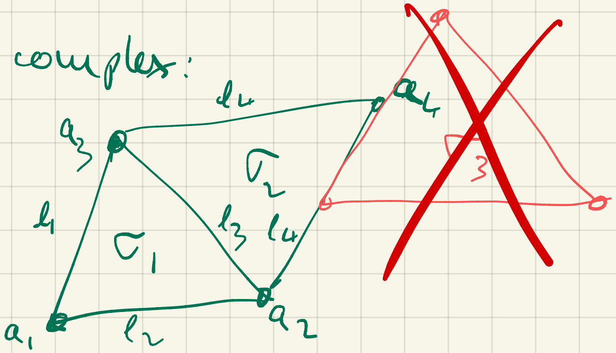
COMPLEXES

A polyhedral complex is any collection K of polyhedra, such that
simplicial complex simplices
fan cones

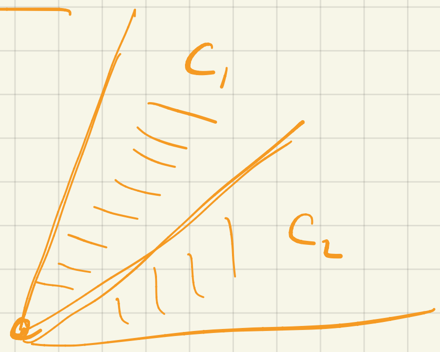
1) K contains all faces of each of its members
2) Any two members of K intersect at a face of both.

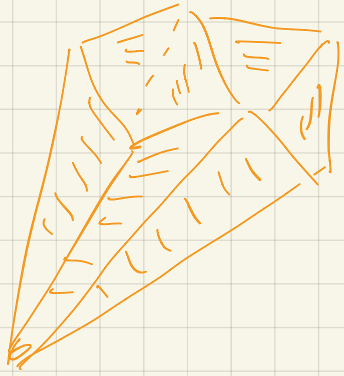
simplex: $\sigma = \mathrm{conv}\{x_0, \dots, x_d\}$, $\dim(\sigma) = d$

Simpl. complex:



$K = \{\sigma_1, \sigma_2, l_1, \dots, l_5, a_1 \dots a_4, \emptyset\}$





COMPLEXES

A polyhedral complex is any collection K of polyhedra, such that

simplicial complex — simplices

fan — cones

1) K contains all faces of each of its members
2) Any two members of K intersect at a face of both.

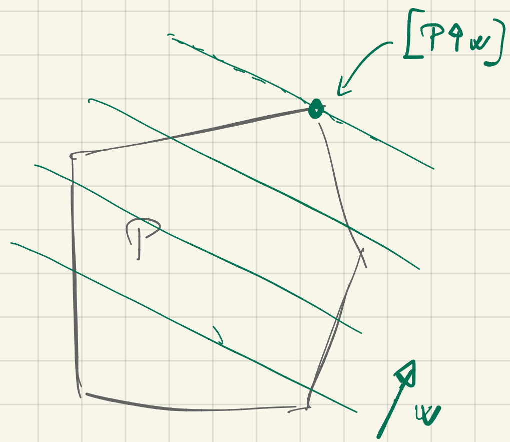
Let K_1 poly. complex in $\mathbb{R}^n$, K_2 poly. cx. in $\mathbb{R}^m$.

A linear isomorphism $K_1 \simeq K_2$ is a linear $\varphi : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ s.t.

the induced map
$$\begin{array}{ccc} K_1 & \longrightarrow & K_2 \\ \rotatebox{90}{$\in$} & & \\ Q & \longmapsto & \varphi(Q) \end{array}$$
is bijective.

NORMAL FANS

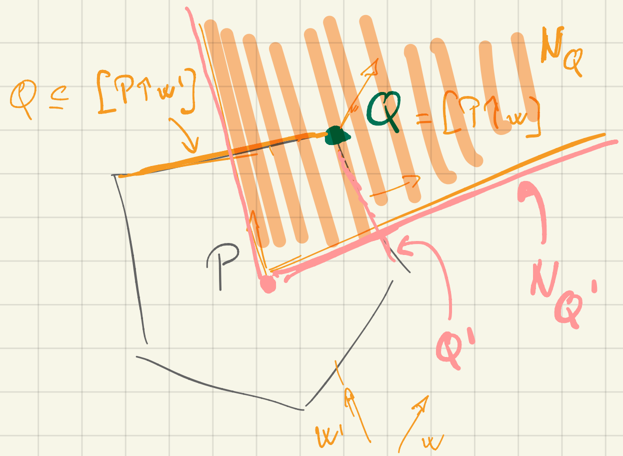
Let P be a polyhedron in $\mathbb{R}^n$ and let $w \in \mathbb{R}^n$.



Define:

$$[P\uparrow w] := \underset{x\in P}{\arg\max}\ \langle x | w\rangle$$

$$= \{x \in P \mid \langle x|w\rangle \geq \langle y|w\rangle \ \forall y \in P\}$$



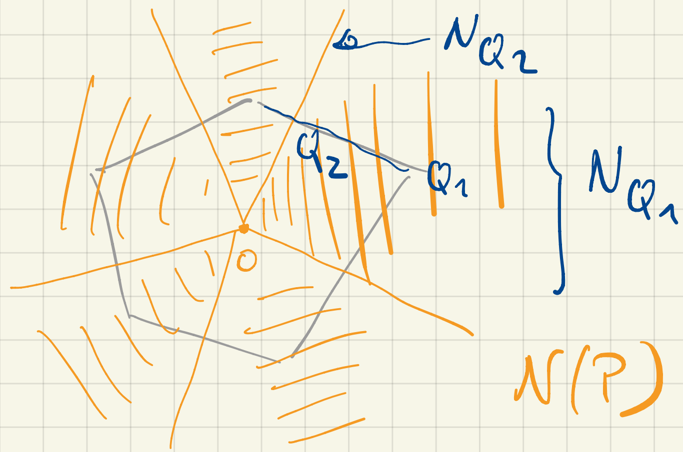
Let P be a polytope, Q any face of P.

Set

$$N_Q := \{w \in \mathbb{R}^n \mid Q \subseteq [P\uparrow w]\}$$

"normal cone"

N_Q is a <u>cone</u>, whose faces are $N_{Q'}$, $Q' \supseteq Q$



$$N(P) = \{N_Q\}_{Q \text{ face of } P}$$

Normal fan to P