WARM-UP 3

(i) Watch the video about polytopes and fans.

(ii) In the notes in Lecture 2 read Proposition 2.13 and its proof.

(iii) Let M be a matroid on the ground set E with set of bases $\mathcal{B}$.

    (a) Let $B \subseteq A \subseteq E$. Determine the set of bases of $M[A]/B$.

    (b) Let $A \subseteq E$ and recall that a loop of M is any $e \in E$ such that $\{e\}$ has rank 0. In other words, $e \in E$ is a loop if it does not appear as element of any basis of M, i.e., if $e \notin B$ for all $B \in \mathcal{B}$.
    Prove that the loops of M/A are exactly the elements of $\mathrm{cl}(A) \setminus A$.